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Research Article

The effect of the flipped classroom model on students' proportional reasoning

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This study aimed to examine the effect of using the flipped classroom model on students' proportional reasoning. In this context, the study focused on students' achievement and solution strategies for proportional reasoning problems. The participants of the study were 56 seventh-grade students, who were determined by convenience sampling method. In line with the purpose of the research, the concept of proportion was taught to the experimental group using the flipped classroom model and to the control group through teaching in accordance with the mathematics curriculum. The data of the study were collected through the Proportional Reasoning Test. The results of the study showed that the flipped classroom model was more effective in terms of mathematics achievement than the teaching method in line with the curriculum. In addition, the experimental group students used more correct solution strategies and fewer incorrect solution strategies than the control group students while solving the problems.

Keywords: Flipped classroom; Middle school; Proportion; Proportional reasoning; Strategies

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1. Introduction

The use of a digital technology approach in education has been recommended by many researchers due to its positive impact on students' learning and understanding (Hwang et al., 2015; National Council of Teachers of Mathematics [NCTM], 2000). This approach has been applied in teaching various subjects, including physics, statistics, computer science, and mathematics (Bergmann & Sams, 2012; Lo & Hew, 2018; Tucker, 2012). Digital technologies make it possible for teachers to record screenshots of a problem or teach a topic from their computer screen and overlay a narrative, create videos of themselves teaching, and curate video lessons from internet sites such as TED-Ed, Khan Academy, or YouTube (Muir & Geiger, 2016). The digital technology approach is becoming increasingly popular, especially in mathematics classrooms.

The digital transformation in the field of education has enabled the Alpha generation, called digital natives, to be intertwined with technology from an early age, shaping their perspectives and worldviews. Just like Generation Z students, Alpha generation students are defined as individuals who can take responsibility for their own learning, have high digital competencies,

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and have an application-oriented learning style (Hussin, 2018). Due to these characteristics, alpha-generation students who perceive technology as a means of connecting with the world need new learning styles (Hernandez-de-Menendez et al., 2020). Integrating technology into education can be seen as a necessity for Alpha generation children to access quality education to ensure their active participation and equality of opportunities (Tondeur, 2012). Moreover, it has been observed that countries that perform well in the Program for International Student Assessment [PISA] and Trends in International Mathematics and Science Study [TIMSS] exams (Mullis et al., 2015; OECD, 2015) have high levels of technology integration in education.

The integration of digital technologies into mathematics education has become increasingly important in the contemporary educational environment (Drijvers & Sinclair, 2023). This is due to the dynamic nature of technology, which has the potential to significantly enhance both teaching and learning experiences in mathematics (Heid, 2005; NCTM, 2000). Digital tools provide a range of interactive resources, such as educational software, online tutorials, and dynamic geometry applications. These resources like concrete materials (Çaylan Ergene & Haser, 2021) can transform abstract mathematical concepts into concrete and engaging experiences for students (Borba, 2016). By appealing to different learning styles and levels of understanding, these technologies make mathematics more accessible and understandable (Hwang et al., 2015). Furthermore, the use of digital technologies can prepare students for a future dominated by technology and enable them to apply mathematical concepts in real-world scenarios that are often technologically driven (Drijvers & Gravemeijer, 2004). Incorporating these tools into mathematics education enriches the learning process and promotes critical thinking, problem-solving skills, and a deeper understanding of mathematical principles (Heid, 2005; Muir & Geiger, 2016). The integration of digital technologies is not only a beneficial development but also a necessary evolution in the field of mathematics education. This necessity enables the use of various online learning methods, such as flipped learning, in the teaching processes of many courses, including mathematics courses.

1.1. The Flipped Classroom

In recent years, the concept of education has shifted from teacher-centered instruction to diverse learner-centered learning modes. This change has resulted in teachers taking on the role of not only knowledge providers but also learning promoters who encourage students to actively construct knowledge. The use of digital technologies is one of the most emphasized and innovative teaching strategies in recent years (Hwang et al., 2015). Digital technologies offer different ways of communicating, accessing information, learning, and teaching (Beatty & Geiger, 2009). digital technologies diversify the content of school mathematics and promote the development of mathematical knowledge and understanding (Heid, 2005). Using digital technology in maths lessons is beneficial for students (Jones, 2000; NCTM, 2000). Moreover, the use of digital technology in mathematics courses is carried out in different ways with different learning models, such as blended and online. The flipped classroom model [FC], one of these teaching models, is a blended learning model that can provide the integration of technology into mathematics education and students' engagement in mathematics (He, 2020; Muir, 2017). In contrast to the traditional method, school work at home and homework at school (Bergmann & Sams, 2012) approach is adopted in FC. FC is an approach based on students learning information outside the classroom with resources such as videos, articles, and lecture notes and doing their homework and activities in the classroom (Tucker, 2012). In FC, teachers can communicate and interact with students online, and deep learning can be supported through face-to-face activities with students in the classroom environment (Cevikbas & Kaiser, 2020).

The FC approach has received increasing attention in recent years as researchers and educators recognize the opportunities that FC can bring to teaching, such as promoting communicative competence and increasing student engagement (Ye, 2023). By flipping the classroom in mathematics courses, educators can create a more dynamic, student-centered learning environment that not only enhances understanding of mathematical concepts but also develops critical thinking and problem-solving skills. Research revealed the positive outcomes obtained with the use of FC in mathematics teaching (Adams & Dove, 2018; Love et al., 2014; Muir, 2017). It has been reported that the use of FC enhances student motivation and interest, as well as increases student-teacher interaction (Bergman & Sams, 2012). In addition, classroom practices have an important place in FC (Eisenhut & Taylor, 2015), and students' engagement in the learning environment increases with active learning strategies (Lugosi & Uribe, 2022), including discussion, small group activities, problem-solving activities used in the classroom (Tucker, 2012). FC increases student achievement (Lai & Hwang, 2016; Wei et al., 2020), and mathematical subjects that are difficult to teach with FC are learned more easily (Muir & Geiger, 2016).In summary, the flipped classroom provides four benefits: active learning, development of students' learning attitudes, efficient use of class time, prioritization of students' learning status, and personal problem-solving (Bergmann & Sams, 2012; Gaughan, 2014).

According to Mason et al. (2013), students' performance in out-of-class learning is significant for both students and teachers during in-class activities. Although FC is an effective alternative teaching and learning strategy, there is not strong enough evidence to conclude whether FC is better than the traditional approach in terms of students' academic performance and perception (Fung et al., 2021). Studies examining the effects of FC on student achievement showed that student achievement in courses conducted with this model was higher than student achievement in courses where traditional methods were used (Bhagat et al., 2016; Day & Foley, 2006) or there was no difference in terms of achievement (Buch & Warren, 2017; Hwang & Lai, 2017). For this reason, there is a need for more experimental research studies using FC (Fung et al., 2021). In this study, the effect of FC on students' proportional reasoning and problem-solving strategies is examined in an experimental setup.

1.2. Proportional Reasoning

One of the aims of mathematics is to provide individuals with mathematical reasoning skills (Van de Walle et al., 2014). There are different mathematical reasoning types: arithmetical, proportional, algebraic, and functional reasoning. All these reasonings are important, but proportional reasoning, which acts as a bridge between arithmetic and algebra in secondary school mathematics, has a special importance (Beckmann & Izsák, 2015). NCTM (2000) emphasizes that proportional reasoning is a form of reasoning that students can use in their professional and daily lives and suggests that students' proportional reasoning skills (hereafter referred to as proportional reasoning) can be developed between the fifth and eighth grades. Proportional reasoning is necessary for meaningful learning of concepts such as ratio and proportion, slope, percentage, and probability (Lesh et al., 1988). According to the Turkish Middle School Mathematics Curriculum (Ministry of National Education [MoNE], 2018), students encounter ratio for the first time at the sixth-grade level and with proportion at the seventh-grade level, and there are also various objectives related to proportion in the curriculum. Three objectives at the sixth-grade level and seven objectives at the seventh-grade level that are directly related to proportional reasoning take place in the curriculum.

Four different types of problems were identified for proportional reasoning: missing-value problems, numerical comparison, qualitative comparison, and inverse proportion (Cramer & Post, 1993; Cramer et al., 1993; Duatepe et al., 2005). Three pieces of numerical information are provided, while one piece is unknown in *missing value problems*. In the problem type of finding the missing value, one of the quantities with a proportional relationship between them is not given, and students are asked to find the unknown value. In *numerical comparison problems*, two ratios are provided, and although a numerical answer is not required, the ratios must be compared. In this kind of problem, students are asked to make a comparison between two ratios defined in the problem. *Qualitative comparison problems* necessitate comparisons that do not depend on specific numerical values. In this type of problem, students are expected to make a comparison between the ratios given verbally without the numerical values given. *Inverse proportion* is a type of problem in which one of the multiplicities that make up the ratio increases while the other decreases at the same rate, or one decreases while the other increases at the same rate.

Students' proportional reasoning levels are determined by considering the interpretations they make to the given proportional situation or the variety of strategies they use (Hines & McMahon, 2005). The correct solution strategies used in solving these problems include unit rate, factor of change, cross-product, and equivalent fractions, while the incorrect solution strategies include additive relationship, emotional response, data omission, and using numbers (Ben-Chaim et al., 1998; Cramer & Post, 1993; Duatepe et al., 2005). In the accessible literature, there exists a limited number of studies in which all problem types for proportional reasoning were used (Duatepe et al., 2005). Moreover, there are studies in which the solution strategies in one or more types of problem are examined (Kahraman et al., 2019; Valverde & Castro, 2012), and there are studies focusing on specific solution strategies (Karlı & Yıldız, 2022). Students' solution strategies in questions measuring proportional reasoning may vary according to question types (Duatepe et al., 2005). Kahraman et al. (2019) found that students mostly used the cross-product strategy and unit rate strategy in numerical comparison problems. Similarly, Valverde and Castro (2012) stated that in missing value problems, the cross-product strategy is mostly used. On the other hand, Karlı and Yıldız (2022) determined that the most commonly used incorrect solution strategy is the additive relationship strategy.

There is a connection between proportional reasoning and many basic concepts in mathematics, science, geography, and art, as well as situations in daily life. In mathematics, proportions, fractions, percentages, similarity, scaling, trigonometry, algebraic equations, measurement, probability, and statistics concepts are related to proportional reasoning. Large-scale international studies such as TIMSS and PISA also consider proportional reasoning as a criterion for students' mathematical competence (Arican, 2019). Even though proportional reasoning is important, students' difficulties in proportional reasoning were reported (Ayan & Isiksal-Bostan, 2018). One of the reasons for these difficulties is placing more emphasis on rule memorization and procedural calculations (Misailadou & Williams, 2003; Modestou & Gagatsis, 2007). Mathematics educators are trying to change their teaching practices to facilitate the development of students' proportional reasoning (Carpenter et al., 1989; Jacobson & Lehrer, 2000). In the accessible literature, a limited number of studies (Ben-Chaim et al., 1998; Çetiner, 2022) were found comparing the traditional teaching method with an alternative teaching method on proportion. In these studies, Ben-Chaim et al. (1998) examined the proportional reasoning of experimental group students taught with the Connected Mathematics Project [CMP] reform curriculum, and Çetiner (2022) examined the proportional reasoning of control group students taught with traditional curriculum. In both studies, experimental group students performed better than control group students. In this study, it was aimed to examine the effect of using the FC on students' proportional reasoning. With this aim, the following research questions guided the present study: How does the proportional reasoning of the seventh-grade students develop through the flipped classroom model?" and "What are the proportional reasoning strategies used by seventh-grade students? Based on the first research question, the following hypotheses were tested:

H0: There is no significant difference in the scores of the Skills-Based Proportional Reasoning Test between the experimental group and the control group.

H1: There is a significant difference in the scores of the Skills-Based Proportional Reasoning Test between the experimental group and the control group.

2. Methodology

In this study, a static group comparative design was used. In this design, a post-test is implemented to the groups, and only the experimental group receives treatment (Creswell, 2018). In line with the purpose of the research, the concept of proportion was taught to the experimental and control groups consisting of seventh-grade students in two different ways: using the FC and teaching in accordance with the mathematics curriculum.

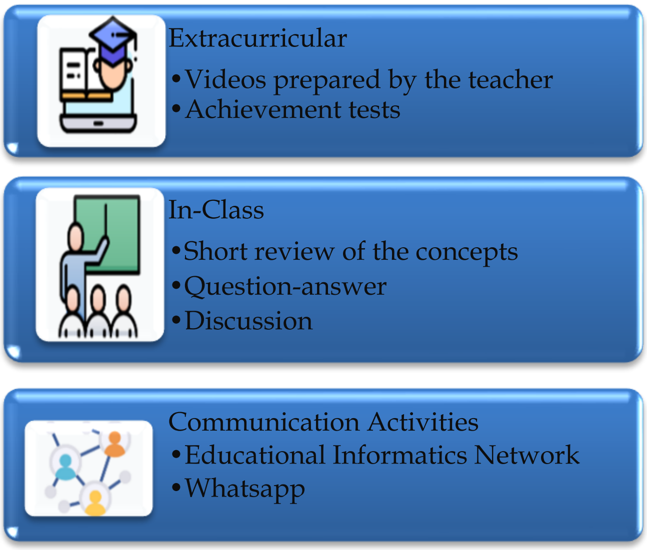
The participants of the study were 56 seventh-grade students (13 years old) (28 students in the experimental group and 28 in the control group) in a middle school in Türkiye, determined by convenience sampling method (Patton, 1990). Although the convenience sampling method may weaken the power of experimental research, it is frequently used in quantitative studies (Etikan et al., 2016). Convenience sampling is an inexpensive and straightforward method of selecting readily available subjects. However, it is crucial for the researcher to clearly explain how this sample differs from a randomly selected one. For this research, our primary objective was to choose a sample suitable for implementing the flipped learning model. This allowed us to gather information about the students in the sample and maintain continuous interaction with the participants throughout the research process. To enhance the validity and reliability of research, it is important to establish a bridge between the researcher and participants (Ergene, 2019). In this study, students were selected from the school where the second researcher was a teacher. The experimental and control groups were selected as equivalent in terms of achievement (Creswell, 2018). As proportional reasoning is a new subject for the participants, a pre-test could not be administered. Therefore, the aim was to select two equivalent groups in terms of mathematics achievement for the experimental and control groups. In this regard, while deciding the experimental group and the control group, the grade points average of five seventh-grade classes in the last five semesters was considered. Then, two classrooms with the closest averages in each semester and with the closest overall average were selected. The willingness of the students in both classrooms to participate in the research and the permission of their parents were determined as criteria. While selecting the experimental group from the two determined classrooms, students' facilities having computer, tablet, or smartphone and internet connection in their homes were taken into account.

2.1. Data Collection Process

The data of the study were collected through the Skills-Based Proportional Reasoning Test [PRT] and PRT Rubric developed by Karaboğaz and Ergene (2023). PRT and PRT Rubric are given in Appendix 1 and Appendix 2. While developing the test, the stages of creating the question pool, expert opinion, pilot study, and statistical analysis specified by Webb (1997) were followed. The item difficulty index values of the problems in the PRT ranged between 0.51 and 0.59, and the corrected-item total correlation values ranged between 0.41 and 0.56. These results showed that the measured values from the items in the PRT were at the desired range (De Vellis, 2003; Pallant, 2007). In the PRT, there were four missing-value problems and two problems in each type of numerical comparison, qualitative comparison, and inverse proportion. Missing value problems can be solved by unit rate, a factor of change, equivalent fraction, cross-product, and increasing strategies. Numerical comparison problems can be solved with unit rate, a factor of change, equivalent fraction, and cross-product strategies. Inverse proportion problems can be solved with the inverse proportion algorithm strategy due to their nature, and other strategies are also used in these problems. In qualitative comparison problems, students are expected to first make sense of the problem and solve it by using different strategies using representations. Student responses to the PRT were scored between 0 and 3 points with the help of the PRT rubric.

The implementation process carried out in the experimental group (Figure 1) consisted of in-class, extracurricular, and communication activities. For extracurricular activities, videos prepared by the teacher and achievement tests were used. In-class activities included a short review of the concepts, question-answer, and discussion activities for the problems that could not be solved in the tests. WhatsApp and Educational Informatics Network [EBA], which is an online social education platform provided free of charge for the use of people in Türkiye, was used for student-student and student-teacher communication. Lecture videos, materials, measurement, and evaluation tools are uploaded to the platform so that students can benefit interactively. The implementation process carried out in the control group consisted of in-class activities and extracurricular activities. In-class activities included lecturing, question-answer, and discussion activities in line with the mathematics curriculum (MoNE, 2018) and solutions to problems related to proportional reasoning. In extracurricular activities, students were given homework, and WhatsApp was used for communication. The time allocated for lessons in the weekly program was longer in the control group than in the experimental group since the control group had in-class lessons. All of the problems in the videos prepared for the experimental group were also solved in the control group during the lesson.

Figure 1
Implementation process carried out with the experimental group



A total of seven videos were produced based on the objectives for the proportion concept in the mathematics curriculum (MoNE, 2018) at the seventh-grade level. The videos were uploaded to the EBA platform weekly. In the videos, teaching of the concept and problem solutions related to the objectives were included. The length of the videos varied between 7' 16'' and 27' 22''. Examples of the problems in the videos are given in Figure 2.

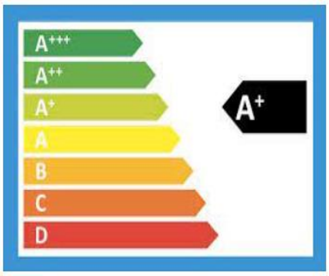
Figure 2
Examples of the proportional reasoning problems in the videos

Doruk goes to the stationery shop opposite the school and sees that the price of four erasers is equal to the price of one pencil. Accordingly, how many erasers can Doruk buy with all of his money if he has enough money to buy 12 pencils?	Danilo Chef added five kilograms of sugar into 7.5 liters of water to make sweet syrup. According to this, how many liters of water per kilogram of sugar is in the syrup?

Students were informed about the FC process, and they were asked to watch the videos meticulously and solve the problems in the videos. The viewing status of the videos on the EBA platform was monitored for each student. The rate of students watching the videos was 98.8%. The students could communicate via both the WhatsApp group and the EBA platform and share and discuss their own ideas and the topics and problems they did not understand on these platforms.

In the in-class activities carried out with the experimental group students, the topics or problems that the students did not understand in the videos were dealt with at the beginning of the lesson. In the rest of the lesson, the researcher solved problems similar to the proportion problems in the seventh-grade mathematics textbook. In the lessons conducted in the experimental and control groups, the same problems were solved in equal numbers using similar solution strategies. Examples of the problems solved in the lessons are given in Figure 3.

Figure 3
Examples of the proportional reasoning problems in the lessons



In the figure on the left, the energy classification of a refrigerator is given. At each level, from top to bottom, the energy consumption of the refrigerator increases by 10 kilowatts per year. The annual energy consumption of a class D refrigerator is 328 kilowatts. Accordingly, how many kilowatts of energy does a person who buys an A+ class refrigerator consume in 1 day? (1 year = 360 days)

Animal hospices are places where stray or abandoned pets usually live. In one hospice, there is enough food for 18 dogs for 40 days. After four days, six more dogs were brought to the hospice. According to this, how many days is the remaining food enough for the dogs in the hospice? (Each dog is given an equal amount of food.)

PRT was implemented to the experimental and control group students after the objectives were covered. The data obtained from the PRT were analyzed in two stages. In the first stage, students' solutions were scored individually by the researchers through the PRT Rubric. An inter-rater reliability was calculated as 98.7% (Miles & Hubermann, 1994) by the researchers for the students' solutions. Skewness and kurtosis coefficients were calculated, and the Shapiro-Wilk test was performed to ensure the normality of the scores obtained from the PRT of the experimental and control groups (see Table 1).

Table 1
Normality values of PRT

				Shapiro-Wilk		
Test	Group	Skewness	Kurtosis	Statistic	SD	*p*
PRT	Experimental	−.102	−1.085	.942	28	.216
	Control	.405	−1.477	.955	28	.266

Table 1 shows that the skewness and kurtosis values of the scores obtained from PRT are between −2 and +2, and hence, the data are suitable for normal distribution (Tabachnick et al., 2013). In cases where the number of participants is less than 50 people, the Shapiro-Wilk test is recommended to test the normality assumption (Razali & Wah, 2011). The Shapiro-Wilk test results reveal that the PRT average scores of the experimental and control groups have a normal distribution at the $\alpha = .05$ significance level. Since the data was found to be normally distributed, an independent sample t-test was used to analyze the data obtained from the experimental and control groups. Moreover, Cohen's d was used to calculate the effect size of the significant difference in the independent sample *t*-test results.

In the second stage of the analysis, the strategies used in solutions to the problems in the PRT were examined. The problems in the test were firstly grouped according to their types as missing-value problems, inverse proportion problems, numerical comparison problems, and qualitative comparison problems. Then, the solutions of the students in the experimental and control groups for these problem types were determined as correct and incorrect solutions. Finally, the solutions of the students for the missing-value problems, inverse proportion problems, and numerical comparison problems in the test were analyzed in the following categories: unit rate strategy, a factor of change strategy, equivalent fraction strategy, equivalence class strategy, a cross-product strategy, additive strategy, inverse proportion algorithm, total relationship strategy, emotional response strategy, data omission strategy, the strategy of using numbers, the strategy of not determining the type of proportion and no answer. In addition, the solutions of the qualitative comparison problems were analyzed in the categories of using visual and numerical values, the existence of proportional reasoning without explicit strategy, and no explicit solution strategy. In the present study, students did not use the equivalence class strategy and data omission strategy. An inter-rater reliability was calculated as 96.5% (Miles & Hubermann, 1994) by the researchers for the strategies used by the students in their solutions.

3. Results

The independent samples *t*-test was conducted to test whether there was a significant difference between the experimental and control groups in terms of the students' PRT scores. The result of the test is presented in Table 2.

Table 2
Independent samples t-test results

Test	*Grup*	*N*	*Mean*	*t*	*SD*	*p*
PRT	Experimental	28	17.93	1.821	54	.010*
	Control	28	13.39			

Note. *$p < .05$

As shown in Table 2, the PRT scores of the experimental group students (*Mean* = 17.93) were higher than those of the control group students (*Mean* = 13.39). Furthermore, the scores of the experimental group students in PRT were 4.54 points higher than those of the control group students, and this difference was statistically significant at p =.05 level (t_{27}=1.821; p =. 010< .05). Thus, the null hypothesis (H0) is rejected, and the alternative hypothesis (H1) is accepted. This indicates a significant difference in PRT scores between the experimental and control groups with a moderate effect size (Cohen d = 0.057) (Cohen, 1988).

3.1. Findings Related to Problem-Solving Strategies

Table 3 provides the strategies used by the experimental group and control group students in PST for each problem type with frequencies and percentages. As can be seen from Table 3, 64% of the experimental group students and 36% of the control group students solved the problems in the PRT correctly. Conversely, 13% of the students did not provide any solutions to the problems, or the solutions were irrelevant. In both groups, the percentage of correct solutions according to the problem types was ranked from high to low as qualitative comparison, numerical comparison, missing value, and inverse proportion.

In the correct solutions, the most frequently used strategies were the cross-product strategy in missing value problems and the unit rate strategy in the numerical comparison problems. In the qualitative comparison problems, the existence of proportional reasoning without explicit strategy, and in the inverse proportion problems, the inverse proportion algorithm strategy was the most frequently used solution strategy. Figure 4 presents the use of the cross-product strategy in solving the missing value problem (4th problem in PRT) while Figure 5 presents the use of the unit rate strategy in solving the numerical comparison problem (6th problem in PRT).

Figure 4
An example of using the cross-product strategy in solving the missing value problem

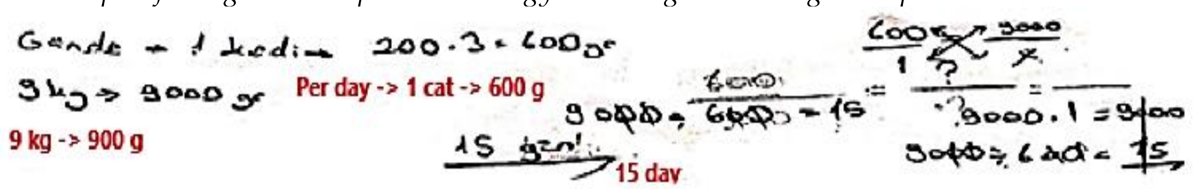


Table 3

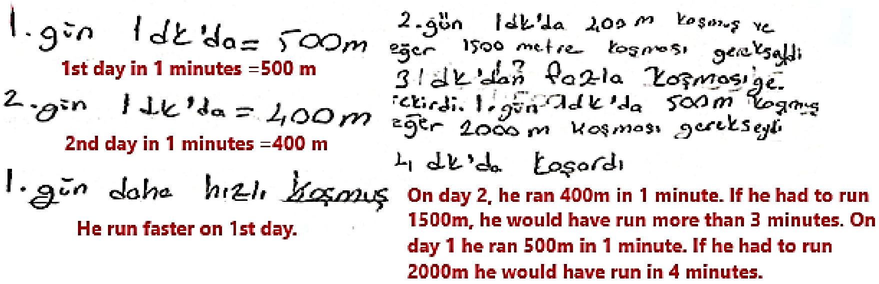
Distribution of Categories Related to the Strategies

	Correct Solution								Incorrect Solution						
	Unit Rate	Factor of Change	Equivalent Fraction	Cross-product	Additive	Inverse Proportion	Using Visual and Numerical Values	The existence of proportional reasoning without explicit strategy	Total Relationship	Using Numerical Values	Emotional Response	Data Omission	No solving strategy	Not determine the type of proportion	No Solution
MV E	5 4%	14 13%	9 8%	34 30%	0 0%	0 0%	0 0%	0 0%	0 0%	36 32%	0 0%	0 0%	0 0%	2 2%	12 11%
MV C	5 4%	10 9%	1 1%	11 10%	9 8%	0 0%	0 0%	0 0%	0 0%	54 48%	0 0%	0 0%	0 0%	2 2%	20 18%
Total	10 4%	24 11%	10 4%	45 20%	9 4%	0 0%	0 0%	0 0%	0 0%	90 40%	0 0%	0 0%	0 0%	4 2%	32 14%
NC E	21 38%	17 30%	1 2%	1 2%	0 0%	0 0%	0 0%	0 0%	1 2%	7 12%	0 0%	0 0%	0 0%	0 0%	8 14%
NC C	11 20%	9 16%	0 0%	0 0%	0 0%	0 0%	0 0%	0 0%	0 0%	17 30%	13 23%	0 0%	0 0%	0 0%	6 11%
Total	32 29%	26 23%	1 1%	1 1%	0 0%	0 0%	0 0%	0 0%	1 1%	24 21%	13 12%	0 0%	0 0%	0 0%	14 13%
QC E	0 0%	0 0%	0 0%	0 0%	0 0%	0 0%	9 16%	33 59%	0 0%	0 0%	0 0%	3 5%	7 13%	0 0%	4 7%
QC C	0 0%	0 0%	0 0%	0 0%	0 0%	0 0%	11 20%	20 36%	0 0%	0 0%	0 0%	0 0%	19 33%	0 0%	6 11%
Total	0 0%	0 0%	0 0%	0 0%	0 0%	0 0%	20 18%	53 47%	0 0%	0 0%	0 0%	3 3%	26 23%	0 0%	10 9%
IP E	0 0%	0 0%	0 0%	0 0%	0 0%	30 54%	0 0%	0 0%	0 0%	7 12%	0 0%	0 0%	0 0%	10 18%	9 16%
IP C	0 0%	0 0%	0 0%	0 0%	0 0%	13 23%	0 0%	0 0%	0 0%	0 0%	0 0%	0 0%	23 41%	11 20%	9 16%
Total	0 0%	0 0%	0 0%	0 0%	0 0%	43 38%	0 0%	0 0%	0 0%	7 6%	0 0%	0 0%	23 21%	21 19%	18 16%
Total E	26 %9	31 11%	10 4%	35 13%	0 0%	30 11%	9 3%	33 12%	1 0%	50 18%	0 0%	3 1%	7 3%	12 4%	33 10%
Total C	16 6%	19 7%	1 0%	11 4%	9 3%	13 5%	11 4%	20 7%	0 0%	71 25%	13 5%	0 0%	42 15%	13 5%	41 14%
Total	42 8%	50 9%	11 2%	46 8%	9 2%	43 8%	20 4%	53 9%	1 0%	121 22%	13 2%	3 1%	49 9%	25 4%	74 13%

Note. E: Experimental group; C: Control Group; MV: Missing value problems; NC: Numerical comparison problems; QC: Qualitative comparison problems; IP: Inverse proportion.

Figure 5

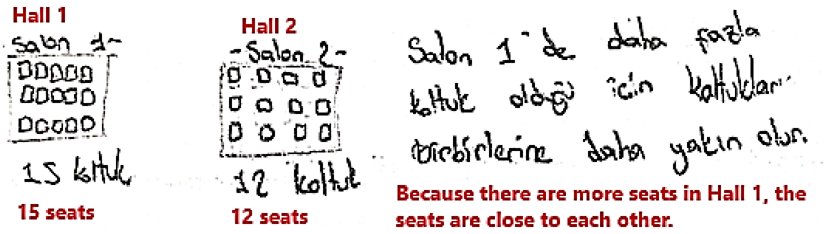
An example of using the unit rate strategy in solving the numerical comparison problem



The categories of strategies for the correctly solved problems in the test demonstrated that the experimental group students used all strategies more than the control group students except using visual and numerical values. The strategies regarding the types of correctly solved problems indicated that the control group students used the increasing strategy in missing-value problems and the visual and numerical values strategy in the qualitative comparison problems. An example of the strategy of using visual and numerical values in the qualitative comparison problem (3rd problem in PRT) is given in Figure 6. In the correctly solved problems, no experimental group student used the increasing strategy, and the number of solutions belonging to the categories of equivalent fraction strategy and using visual and numerical values was low.

Figure 6

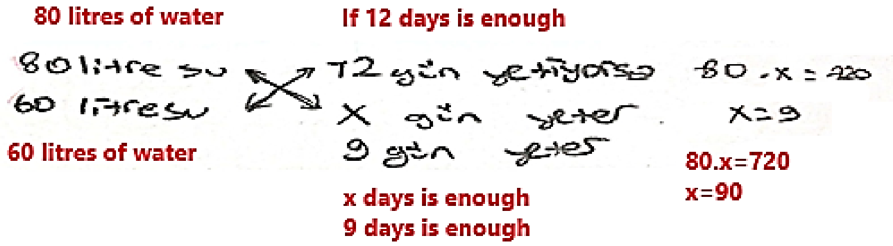
An example of using the visual and numerical values strategy in solving the qualitative comparison problem



In the categories of strategies for incorrect solutions, the using numbers strategy was the most frequently used strategy in the missing-value problems and numerical comparison problems. In 23% of the solutions of the qualitative comparison problems and 21% of the solutions of the inverse proportion problems, a clear solution strategy was not observed. Moreover, students could not determine the type of proportion in 19% of the solutions of the inverse proportion problems. An example of the failure to determine the type of proportion in the inverse proportion problem (8th problem in PRT) is given in Figure 6. The emotional response strategy was used only by the control group students in solving numerical proportion problems.

Figure 6

Example of a strategy of not determining the type of proportion in solving the inverse proportion



4. Discussion and Conclusion

This study examined how the use of FC, which is characterized as a different model in teaching, affects students' proportional reasoning. In this context, the study focused on students' achievement and solution strategies for proportional reasoning problems. The participants of the research consisted of the experimental group students in whom FC was used in teaching proportion and the control group students who were taught in accordance with the curriculum. In the study, the same problems were solved by using the same solution strategies for the experimental and control group students. The data of the study were collected with the PRT Karaboğaz and Ergene (2023), which includes problems belonging to all types of proportional reasoning problems.

In the study, there was a statistically significant difference between the PRT post-test scores of the experimental group and control group students (MoNE, 2018) (t_{27}=1.821; p = .010<.05), and this difference is significant and moderately effective (Cohen d = .057) in favor of the experimental group. This finding suggests that the FC was more effective in terms of mathematics achievement than the teaching method in line with the curriculum. Accordingly, the FC enabled students to solve more proportional reasoning problems correctly and thus to be more successful in the context of proportional reasoning problems. Studies comparing the FC and the traditional classroom model in mathematics lessons generally revealed that the FC was more effective in student achievement (Adams & Dove, 2018; Bhagat et al., 2016; Lo & Hew, 2018; Love et al., 2014; Muir, 2017). This result of the study is consistent with the studies comparing the traditional teaching method with an alternative teaching method on proportion (Ben-Chaim et al., 1998; Çetiner, 2022). This may be explained by the fact that traditional teaching methods emphasizing rule memorization and procedural calculations cause students to have difficulty in proportional reasoning (Misailadou & Williams, 2003; Modestou & Gagatsis, 2007). This study revealed that the FC can be used as a beneficial and different teaching model while teaching proportional reasoning.

Sixty-four percent of the experimental group students and 36% of the control group students solved the problems in the PST correctly. However, the correct answer rates of the experimental group and control group students according to the problem types were listed as qualitative comparison, numerical comparison, missing value, and inverse proportion problems from high to low. Consequently, FC did not affect the order in which the proportional reasoning problem types were answered from high to low, but it increased student achievement in these problem types. Thus, the experimental group students were more successful than the control group students in all proportional reasoning problem types, which means that FC was more effective than teaching according to the mathematics curriculum in student achievement for each problem type. Considering that proportional reasoning problem types are an important factor affecting students' performance (Alatorre & Figueras, 2004; Lawton, 1993), it can be stated that FC is a teaching model that increases success for each problem type.

Students' proportional reasoning levels are determined by considering the variety of strategies they use in the problem for the given proportional situation (Hines & McMahon, 2005). Therefore, giving importance to the strategy used and process-oriented approach rather than evaluating the correct result in proportional reasoning problem solutions is crucial (Kahraman et al., 2019). In the present study, students used eleven different strategies in solving proportional reasoning problems. While seven of these strategies were correct solution strategies, four of them were incorrect solution strategies. Hence, the proportional reasoning test used in the study was suitable for students to use different strategies (Karaboğaz & Ergene, 2023). As Slovin (2000) stated, in order to improve students' proportional reasoning, the context used in proportion problems should be conducive to the use of different strategies rather than the traditional approach.

In the study, the most frequently used correct answer strategies were the cross-product strategy in the missing-value problems and the unit rate strategy in the numerical comparison problems. This finding is consistent with that of Akkuş-Çıkla and Duatepe (2002), who found the most frequently used strategies were the cross-product strategy in missing-value problems, and that of

Kahraman et al. (2019), who found the most frequently used strategies were the unit rate strategy in numerical comparison problems (Kahraman et al., 2019). In the inverse proportion problems, the inverse proportion algorithm was the most used correct answer strategy. This outcome is contrary to previous studies, which found that the cross-product strategy was the most used strategy in inverse proportion problems (Duatepe et al., 2005). The use of a different strategy in the inverse proportion problem type can be considered as an indicator that students' proportional reasoning levels have increased (Hines & McMahon, 2005).

The experimental group students used fewer incorrect solution strategies than the control group students in solving the problems in the PRT. In the missing value problems and numerical comparison problems, the strategy of using numbers and in the inverse proportion problems, the failure to determine the type of proportion were the most commonly used incorrect solution strategies. This finding is consistent with the previous research studies in the literature (Duatepe et al., 2005; Karlı & Yıldız, 2022). In addition, the control group students left more problems unanswered, made incorrect solutions without using a solution strategy and used the emotional response strategy more than the experimental group students. In the emotional response strategy, subjective answers that are not mathematical are given by associating with real life. This strategy is used when proportional reasoning skills are not developed (Ben-Chaim et al., 1998). No use of his strategy in the experimental group supports the idea that students showed proportional reasoning with the FC.

The data of this study is limited to students' solution to the PRT. In addition, problem-posing and problem-solving skills (Ergene, 2022; Ergene & Çaylan Ergene, 2023) related to proportional reasoning can be handled together. Moreover, there is no qualitative data on students' thoughts about the FC and why they provided incorrect solutions to proportional reasoning problems. It is suggested that qualitative data be included in future research studies that adopt this research design. In conclusion, FC was effective in the development of proportional reasoning. Therefore, it is suggested that FC should be applied to other types of reasoning, such as arithmetical, algebraic, and functional reasoning.

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References

Adams, C., & Dove, A. (2018). Calculus students flipped out: The impact of flipped learning on calculus students' achievement and perceptions of learning. *PRIMUS, 28*(6), 600–615. https://doi.org/10.1080/10511970.2017.1332701

Akkus-Çıkla, O., & Duatepe, A. (2002). A qualitative study on the proportional reasoning skills of the preservice elementary mathematics teachers. *Hacettepe University Journal of Education. 23*(23), 32–40.

Alatorre, S. & Figueras, O. (2004). Proportional reasoning of quasi-illit-erate adults. In M. J. Hoines & A. B. Fuglestad (Eds.), *Proceedings of the 28th conference of the international group for the psychology of mathematics education* (Vol. 2, pp. 9–16). Bergen, Norway

Arican, M. (2019). A diagnostic assessment to middle school students' proportional reasoning. *Turkish Journal of Education*, *8*(4), 237–257. https://doi.org/10.19128/turje.522839

Ayan, R., & Isiksal-Bostan, M. (2018). Middle school students' proportional reasoning in real life contexts in the domain of geometry and measurement. *International Journal of Mathematical Education in Science and Technology*, *50*(1), 65-81. https://doi.org/10.1080/0020739X.2018.1468042

Beatty, R., & Geiger, V. (2010). Technology, communication, and collaboration: Re-thinking com-munities of inquiry, learning and practice. In C. Hoyles, & J. B. Lagrange (Eds.), *Mathematics education and technology – rethinking the terrain* (pp. 251–284). Springer. https://doi.org/10.1007/978-1-4419-0146-0_11

Beckmann, S., & Izsák, A. (2015). Two perspectives on proportional relationships: Extending complementary origins of multiplication in terms of quantities. *Journal for Research in Mathematics Education, 46*(1), 17–38. https://doi.org/10.5951/jresematheduc.46.1.0017

Ben-Chaim, D., Fey, J. T., Fitzgerald, W. M., Benedetto, C. ve Miller, J. (1998). Proportional reasoning among 7th grade students with different curricular experiences. *Educational Studies in Mathematics, 36*(36), 247–273. http://dx.doi.org/10.1023/A:1003235712092

Bergmann, J., & Sams, A. (2012). *Flip your classroom: Reach every student in every class every day*. International Society for Technology in Education.

Bhagat, K. K., Chang, C. N., & Chang, C. Y. (2016). The impact of the flipped classroom on mathematics concept learning in high school. *Journal of Educational Technology & Society, 19*(3), 134–142. https://www.jstor.org/stable/jeductechsoci.19.3.134

Borba, M. C., Askar, P., Engelbrecht, J., Gadanidis, G., Llinares, S., & Aguilar, M. S. (2016). Blended learning, e-learning and mobile learning in mathematics education. *ZDM Mathematics Education, 48*, 589–610. https://doi.org/10.1007/s11858-016-0798-4

Buch, G. R., & Warren, C. B. (2017). The Flipped Classroom: Implementing Technology to Aid in College Mathematics Student's Success. *Contemporary Issues in Education Research, 10*(2), 109–116. https://doi.org/10.19030/cier.v10i2.9921

Carpenter, T., Fennema, E., Peterson, P., Chiang, C., & Loef, M. (1989). Using knowledge of children's mathematical thinking in classroom teaching: Anexperimental Study. *American Educational Research Journal, 26*, 499–531. https://doi.org/10.3102/00028312026004499

Cevikbas, M., Kaiser, G. (2020). Flipped classroom as a reform-oriented approach to teaching mathematics. *ZDM Mathematics Education 52*, 1291–1305. https://doi.org/10.1007/s11858-020-01191-5

Cohen, J. (1988). *Statistical power analysis for the behavioural sciences*. Routledge.

Cramer, K. & Post, T. (1993). Connecting research to teaching proportional reasoning. *Mathematics Teacher, 86*(5), 404–407. https://doi.org/10.5951/MT.86.5.0404

Cramer, K., Post, T., & Currier, S. (1993). Learning and teaching ratio and proportion: Research implications. In D. Owens (Ed.), *Research ideas for the classroom* (pp. 159–178). Macmillan Publishing Company.

Creswell, J. W. (2018). *Research design: Qualitative, quantitative, and mixed methods approaches*. Sage.

Çaylan-Ergene, B., & Haser, Ç. (2021). Students' algebra achievement, algebraic thinking and views in the case of using algebra tiles in groups. *Necatibey Faculty of Education Electronic Journal of Science and Mathematics Education, 15*(2), 254–281. https://doi.org/10.17522/balikesirnef.1019292

Çetiner, S. (2022). *The effect of concept cartoons the proportional reasoning skills of 7th grade students* (Publication no. 736424) [Doctoral dissertation, Marmara University]. Council of Higher Education Thesis Center.

Day, J. A. ve Foley, J. D. (2006). Evaluating a web lecture intervention in a human– computer interaction course. *IEEE Transactions on Education, 49*(4), 420–431. https://doi.org/10.1109/TE.2006.879792

De Vellis, R. F. (2003). *Scale development: Theory and application*. Sage Publications.

Drijvers, P., & Gravemeijer, K. (2005). Computer algebra as an instrument: Examples of algebraic schemes. In D. Guin, K. Ruthven, & L. Trouche (Eds.), *The didactical challenge of symbolic calculators: Turning a computational device into a mathematical instrument* (pp. 163–196). Kluwer.

Drijvers, P., & Sinclair, N. (2023). The role of digital technologies in mathematics education: purposes and perspectives. *ZDM–Mathematics Education*, 1–10. https://doi.org/10.1007/s11858-023-01535-x

Duatepe, A., Akkuş-Çıkla, O., & Kayhan, M. (2005). An investigation on students' solution strategies for different proportional reasoning items. *Hacettepe University Journal of Education, 28*, 73–81.

Eisenhut, L. A., & Taylor, C. E. (2015). In-class purposes of flipped mathematics educators. *Journal of Mathematics Education at Teachers College, 6*(2), 17–25. http://dx.doi.org/10.7916/jmetc.v6i2.615

Ergene, Ö. (2019). *Investigation of pre-service mathematics teachers' understanding of definite integral through modelling by using Riemann sums*. Unpublished doctoral dissertation. Marmara University.

Ergene, Ö. (2022). Posing probability problems related to continuous and discrete sample space. *International Journal of Mathematical Education in Science and Technology, 53*(2), 311–336. https://doi.org/10.1080/0020739X.2021.2004464

Ergene, Ö., & Çaylan Ergene, B. (2023). Posing problems and solving self-generated problems: The case of convergence and divergence of series. *International Journal of Mathematical Education in Science and Technology*. https://doi.org/10.1080/0020739X.2023.2170292

Etikan, I., Musa, S. A., & Alkassim, R. S. (2016). Comparison of convenience sampling and purposive sampling. *American Journal of Theoretical and Applied Statistics, 5*(1), 1–4.

Fung, C. H., Besser, M., & Poon, K. K. (2021). Systematic literature review of flipped classroom in Mathematics. *Eurasia Journal of Mathematics, Science and Technology Education*, *17*(6), 10900. https://doi.org/10.29333/ejmste/10900

Gaughan, J. E. (2014). The flipped classroom in world history. *The History Teacher, 47*(2), 221–244. https://www.jstor.org/stable/43264225

He, J. (2020). Research and practice of flipped classroom teaching mode based on guidance case. *Education and Information Technologies, 25*, 2337–2352. https://doi.org/10.1007/s10639-020-10137-z

Heid, K. (2005). Technology in mathematics education: tapping into visions of the future. In W. J. Masalski (Ed.), *Technology-Supported Mathematics Learning Environments*. NCTM.

Hernandez-de-Menendez, M., Díaz, C. A. E., & Morales-Menendez, R. (2020). Educational experiences with Generation Z. *International Journal on Interactive Design and Manufacturing (IJIDeM), 14*(3), 847–859. https://doi.org/10.1007/s12008-020-00674-9

Hines, E., & McMahon, M. T. (2005). Interpreting middle school students proportional reasoning strategies: *Observations from preservice teachers. School Science and Mathematics, 105*(2), 88–105. https://doi.org/10.1111/j.1949-8594.2005.tb18041.x

Hussin, A. A. (2018). Education 4.0 made simple: Ideas for teaching. *International Journal of Education and Literacy Studies, 6*(3), 92–98. https://doi.org/10.7575/aiac.ijels.v.6n.3p.92

Hwang, G.J., & Lai, C.L. (2017). Facilitating and Bridging Out-Of-Class and In-Class Learning: An Interactive E-Book-Based Flipped Learning Approach for Math Courses. *Educational Technology and Society, 20*(1), 184–197. https://www.jstor.org/stable/jeductechsoci.20.1.184

Hwang, G.J., Lai, C.L. & Wang, S.Y. (2015). Seamless flipped learning: a mobile technology-enhanced flipped classroom with effective learning strategies. *Journal of Computers in Education, 2*(4), 449–473. https://doi.org/10.1007/s40692-015-0043-0

Jacobson, C., & Lehrer, R. (2000). Teacher appropriation and student learning of geometry through design. *Journal for Research in Mathematics Education, 31*(31), 71–88. https://doi.org/10.2307/749820

Jones, K. (2000). Providing a foundation for deductive reasoning: Students' interpretations when using Dynamic Geometry software and their evolving mathematical explanations. *Educational Studies in Mathematics, 44*(1-2), 55–85. https://doi.org/10.1023/A:1012789201736

Kahraman, H., Kul, E., & Aydoğdu İskenderoglu, T. (2019). Strategies Employed by 7th and 8th Graders for Quantitative Proportional Reasoning Problems *Turkish Journal of Computer and Mathematics Education (TURCOMAT), 10*(1), 195–216. https://doi.org/10.16949/turkbilmat.333046

Karaboğaz, Y., & Ergene, Ö. (2023). Development of a skillsbased proportional reasoning test. *Journal of Individual Differences in Education, 5*(1), 31–47. https://doi.org/10.47156/jide.1293584

Karlı, M. G., & Yıldız, E. (2022). Incorrect Strategies Developed by Seventh-Grade Students to Solve Proportional Reasoning Problems. *Journal of Qualitative Research in Education, 29*, 111–148. https://doi.org/10.14689/enad.29.5

Lai, C. L., & Hwang, G. J. (2016). A self-regulated flipped classroom approach to improving students' learning performance in a mathematics course. *Computers & Education, 100*, 126–140. https://doi.org/10.1016/j.compedu.2016.05.006

Lawton, C. A. (1993). Contextual factors affecting errors in proportional reasoning. *Journal for Research in Mathematics Education, 24*(5), 460–466. https://www.jstor.org/stable/749154

Lesh, R., Post, T. & Behr, M. (1988). Proportional reasoning. In J. Hiebert & M. Behr (Eds.), *Number concepts and operations in the middle grades* (pp. 93–118). National Council of Teachers of Mathematics.

Lo, C. K., & Hew, K. F. (2018). A comparison of flipped learning with gamification, traditional learning, and online independent study: the effects on students' mathematics achievement and cognitive engagement. *Interactive Learning Environments*, *28*(4), 464-481. https://doi.org/10.1080/10494820.2018.1541910

Love, B., Hodge, A., Grandgenett, N., & Swift, A. W. (2014). Student learning and perceptions in a flipped linear algebra course. *International Journal of Mathematical Education in Science and Technology, 45*(3), 317–324. https://doi.org/10.1080/0020739X.2013.822582

Lugosi, E., & Uribe, G. (2022). Active learning strategies with positive effects on students' achievements in undergraduate mathematics education. *International Journal of Mathematical Education in Science and Technology, 53*(2), 403–424. https://doi.org/10.1080/0020739X.2020.1773555

Mason, G. S., Shuman, T. R., & Cook, K. E. (2013). Comparing the effectiveness of an inverted classroom to a traditional classroom in an upper-division engineering course. *IEEE Transactions on Education, 56*(4), 430e435.

Miles, B. M., & Huberman, A. M. (1994). *Qualitative data analysis.* Sage.

Ministry of National Education [MoNE]. (2018). *Matematik dersi öğretim programı: İlkokul ve Ortaokul 1, 2, 3, 4, 5, 6, 7 ve 8. Sınıflar* [Mathematics curriculum: Primary and Secondary School 1st, 2nd, 3rd, 4th, 5th, 6th, 7th and 8th Grades]. Author.

Misailadou, C., & Williams, J. (2003). Measuring children's proportional reasoning, the "tendency" for an additive strategy and the effect of models. In N. A. Pateman, B. J. Dougherty, & J. T. Zilliox (Eds.), *Proceedings of the 27th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 293–300). University of Hawaii.

Modestou, M., & Gagatsis, A. (2007). Students' improper proportional reasoning: A result of the epistemological obstacle of "linearity." *Educational Psychology, 27*(1), 75–92. https://doi.org/10.1080/01443410601061462

Muir, T. (2017). Flipping the mathematics classroom: affordances and motivating factors. *The Mathematics Educator, 17*(1&2), 105–130.

Muir, T., & Geiger, V. (2016). The affordances of using a flipped classroom approach in the teaching of mathematics: a case study of a grade 10 mathematics class. *Mathematics Education Research Journal, 28*, 149–171. https://doi.org/10.1007/s13394-015-0165-8

Mullis, I. V., Martin, M. O., Foy, P., & Hooper, M. (2015). *TIMSS 2015 International Results in Mathematics.* IEA TIMMS & PIRLS.

National Council of Teachers of Mathematics [NCTM]. (2000). *Principles and Standards for School Mathematics.* Author.

OECD. (2015). *Immigrant students at school: Easing the journey toward integration*. Author.

Pallant J. (2007). SPSS *Survival manual: A step by step guide to data analysis using SPSS*. Allen & Unwin.

Patton, M. Q. (1990). *Qualitative evaluation and research methods.* Sage.

Razali, N. M. & Wah, Y. B. (2011). Power comparisons of Shapiro-Wilk, Kolmogorov-Smirnov, Lilliefors and Anderson-Darling tests. *Journal of Statistical Modeling and Analytics, 2*(2), 21–33.

Slovin, H. (2000). Take Time for Action: Moving to Proportional Reasoning. *Mathematics Teaching in the Middle School, 6*(1), 58–60. https://doi.org/10.5951/MTMS.6.1.0058

Tabachnick, B. G., Fidell, L. S., & Ullman, J. B. (2013). *Using multivariate statistics*. Pearson.

Tondeur, J., Van Braak, J., Sang, G., Voogt, J., Fisser, P., & Ottenbreit-Leftwich, A. (2012). Preparing pre-service teachers to integrate technology in education: A synthesis of qualitative evidence. *Computers & Education, 59*(1), 134–144. https://doi.org/10.1016/j.compedu.2011.10.009

Tucker, B. (2012). The flipped classroom. *Education Next*, *12*(1), 82–83.

Valverde, G., & Castro, E. (2012). Prospective elementary school teachers' proportional reasoning. *PNA, 7*(1), 1–19.

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2014). *Elementary and middle school mathematics*. Pearson.

Webb, N. L. (1997). *Determining alignment of expectations and assessments in mathematics and science education.* National Institute for Science Education.

Wei, X., Cheng, I. L., Chen, N. S., Yang, X., Liu, Y., Dong, Y., Zhai, X., & Kinshuk. (2020). Effect of the flipped classroom on the mathematics performance of middle school students. *Educational Technology Research and Development, 68*, 1461–1484. http://dx.doi.org/10.1007/s11423-020-09752-x

Ye, X. (2023). Insiders' perceptions of the flipped classroom approach in Chinese EFL junior secondary schools. *Education 3-13. 51*(6), 988–1002. https://doi.org/10.1080/03004279.2022.2033293

Appendix 1. Skills-Based Proportional Reasoning Test

1) Brisk walking for 1 hour a day burns an average of 300 calories. Knowing that regular brisk walking is beneficial for a healthy life, Ali walks every day. After dinner, Ali ate three slices of baklava and drank 200 ml of lemonade. The calorie values of the baklava and lemonade are given below. Ali thought that what he ate was more than the calories he needed for the day, and he went for a brisk walk to spend the calories he got from the eating**. Accordingly, how many hours of brisk walking would Ali need to do in order to burn the extra calories?**

1 Slices Baklava
175 Calories

100 ml Lemonade
37.5 Calories

2) Due to global warming and unconscious water consumption, groundwater and surface waters are decreasing. This situation makes it mandatory for people to consume water more consciously.

A family with a water tank on the roof of their house uses 80 liters of water per day, and all water in the tank is enough for 12 days. Levent, the youngest son of the family, who saw the poster about water conservation, was very impressed by the image and searched and explained to his family what could be done to use water consciously.

After that, the amount of water used by the family per day became 60 liters. **According to this, how many days is the water in the full water tank enough for the family?**

3) Seats with the same characteristics are installed in Hall 1 and Hall 2 of two movie theatres having the same area. Since the number of seats in Hall 2 is less than the number of seats in Hall 1, in which hall are the seats installed close to each other?

Explain your answers in detail.

4) Erdem received a project assignment from the mathematics course, and his teacher asked him to use his knowledge of mathematics and geometry to design a machine that could be useful in daily life. Erdem, who has a great interest in and love for stray animals, designed the feeding machine, which he called the "Smart Feeder," where he could feed these animals. While making a presentation to his classmates, Erdem said, "The tank of the smart feeding machine I designed can be filled with 9 kg of dry cat food, and 200 g of feed is automatically poured into the feeder every morning, noon, and evening in a day.".

Accordingly, how many days is the feed in the Smart Feeder with a full tank enough for stray cats?

5) Information: Since the gravity on the Moon is the same as on Earth, a person with a mass of 60 kg on Earth would have a mass of 10 kg on the Moon.

NASA (National Aeronautics and Space Administration) has set some conditions to determine the people who will travel to the Moon in 2025. One of these conditions is that the mass of the people to be sent to the Moon should not exceed 12.5 kilograms. **The weights of the four applicants who met the other conditions set by NASA for the trip to the Moon are given in the table below. According to this, which person or persons should NASA reject? Explain**.

Applicants	Mass in the World
Efsa	66 kg
Nisa Zeynep	72 kg
Omer	76 kg
Burak	69 kg

6) Participating in a running competition, Ahmet ran 1500 meters in three minutes on the first day and 2000 meters in five minutes on the second day.

According to this, which day did Ahmet run faster? Explain

Appendix 1 continued

7) Natural gas meters used in residences are of two types: postpaid and card meters. Customers who use card meters can top up their natural gas cards with the amount of money or cubic meters they wish with the machine located at certain points. Since the weather did not get too cold in October, Ayşe paid 750 TL to the gas meter and loaded 125 cubic meters of gas onto her gas card.
When the weather gets colder in November, Ayşe feels the need to buy more gas. If Ayşe pays how much TL to the gas machine, 240 cubic meters will be loaded onto her card?

8) A fully filled carboy is placed in a water dispenser. With all the water in this carboy, a maximum of 95 glasses of water can be filled so that 200 ml plastic cups are full.
According to this, at most, how many 500 ml water bottles can be fully filled from a fully filled carboy placed in the dispenser?

9) Global climate changes are being experienced due to the increase in the level of carbon dioxide gas accumulating in the atmosphere. In this context, many automobile-producing countries are switching to the production of electric cars instead of cars that have high carbon dioxide emissions and run on fossil fuels (e.g., gasoline and diesel). In our country, the production of electric cars has started, and in the promotions made, it was stated that one of the cars to be produced will travel 500 km with 100% charge. Ata, who wants to buy this electric vehicle, calculates that only 4% of the vehicle's charge will be used up when he goes from home to work and back home again. According to this, how many kilometers is the distance between Ata's home and workplace?

10) Ergin drank tea after lunch in a smaller glass and with more sugar than the tea he drank at breakfast. The taste of the tea Ergin drank after lunch compared to the taste of the tea he drank at breakfast in the morning is a) More tasteless, b) More sweet, c) Tastes the same, and d) The information given is insufficient.
Explain which of the above options is correct.

Appendix 2. Skills-Based Proportional Reasoning Test Rubric

Missing Value Problems (1st, 4th ,7th and 9th problem) **Inverse Proportion Problems (2nd and 8th problem)**	**0 Point** • No solution • No evidence that proportional reasoning exists • There is an additive comparison of the data • There is a randomization of data, numbers, and operations **1 Point** • Only the result is indicated •There is evidence that proportional reasoning exists (such as establishing proportions between incorrect variables or using visual data) • Proportion type is not recognized **2 Point** • There is proportional reasoning between expected variables, but no result is reached • There is proportional reasoning between expected variables, but there is a calculation error **3 Point** • There is proportional reasoning required to solve the problem completely and correctly, and the result is reached
Numerical Comparison Problems (5th and 6th problem)	**0 Point** • No solution • Only the result is indicated • Incorrect proportion between variables • No evidence that proportional reasoning exists • There is an additive comparison of the data • There is a randomization of data, numbers, and operations **1 Point** • By using or not using proportional reasoning between the expected variables, the correct conclusion is reached but misinterpreted • Correct answer but insufficient explanation **2 Point** • Proportional reasoning is shown between the expected variables; the explanation was insufficient, or the correct result could not be reached due to a calculation error **3 Point** • Although the correct result is not reached, the correct interpretation is made according to the result found • The proportional reasoning required to reach the correct result is demonstrated at a good level, and the correct explanation is made
Qualitative comparison problems (3rd and 10th problem)	**0 Point** • No solution • No evidence that proportional reasoning exists • Only the correct answer is marked, no explanation **1 Point** • Only one of the data in the problem is used to result, and the correct answer was marked **2 Point** • Correct answer is marked; incorrect or incomplete explanation is made by using both of the data in the problem **3 Point** • The expected correct answer is found, and the explanation is made using the expressions in the problem • The expected correct answer is found, the explanation is made with original sentences, not using the expressions in the problem, and the explanations are enriched with methods such as creating shapes, drawing, and giving examples